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N-FREQUENCY UNDULATOR

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Abstract

Free-electron lasers (FEL) imply the elaboration of insertion devices. Generation of synchrotron radiation uses storage rings and free electrons laser components. In this type of insertion devices the motion of relativistic electrons generates tunable coherent radiation with desired wavelength. A beam of relativistic electrons undergo transverse oscillation and emit radiation. In FEL research and development one of the main trends is the elaboration of the compact devices. The undulator is the principal component where the phenomenon of coherent radiation takes place. The quality of the beam and of the resulting radiation can be improved by trying different field patterns, coil pitches, dimensions (tapering). The models evolved from single-frequency longitudinal scheme to two-frequency transversal, then to three-frequency transversal and now with a general form of N-frequency model. This expansion form a single frequency to multiple frequencies responds to a practical need. Many applications, in diverse areas as biology, pharmacy, chemistry, metamaterials and others, require several beams with different frequencies in order to get selective interactions with the matter. A new undulator magnetic structure for free electron lasers is presented. For the N-frequency undulator and injection on the longitudinal axis, the trajectory of electrons is computed. This structure reduces the peak value of the magnetic field component.

Key words: *FEL, magnetic field, coherent radiation*

Introduction

Free-electron lasers (FEL) imply the elaboration of insertion devices. Generation of synchrotron radiation uses storage rings and free electrons laser components. In this type of insertion devices the motion of relativistic electrons generates tunable coherent radiation with desired wavelength [1–7]. A beam of relativistic electrons undergo transverse oscillation and emit radiation. The radiation field and the electron wiggles velocity further produces axial ponderomotive force and bunch the electrons along the length of the undulator [1, 4].

In FEL research and development one of the main trends is the elaboration of the compact devices. The undulator is the principal component where the phenomenon of coherent radiation takes place. The quality of the beam and of the resulting radiation can be improved by trying different field patterns, coil pitches, dimensions (tapering). The models evolved from single-frequency longitudinal scheme [2] to two-frequency transversal [3], to three-frequency transversal [4, 5]. This expansion form a single frequency to multiple frequencies responds to a practical need. Many applications, in diverse areas as biology, pharmacy, chemistry, metamaterials and others, require several beams with different frequencies in order to get selective interactions with the matter.

Equation of Motion and Radiation in an Undulator or Wiggler

This paper extends the theory of the two-frequency [3], three-frequency [4] and improved models [5] to a model including N frequencies. We assume that the electron moves on-axis in a N-harmonic undulator structure, with magnetic field given by:

$$\vec{B} = [B_x, B_z, B_s] \text{and } B_x = 0, B_z = \sum_{h=1}^{N} B_0 a_h \sin(k_h z) \ , \tag{1}$$

where $k_u = 2\pi / \lambda_u$, $k_h = h\ k_u$, λ_u is the undulator wavelength, B_0 is peak value of the field strength, h is an integer, and a_h, $h = 1, \dots, N$ are parameters.

The electron trajectory is determined by the Lorentz force:

$$\vec{F} = m_0 \gamma \dot{\vec{v}} = e \vec{v} \times \vec{B} \ . \tag{2}$$

A straightforward calculation yields for the velocity components:

$$\dot{v}_x = \frac{e}{m_0 \gamma}(-v_s B_z),\ \dot{v}_z = \frac{e}{m_0 \gamma}(-v_x B_s),\ \dot{v}_s = \frac{e}{m_0 \gamma}(v_x B_z) \ . \tag{3}$$

Since the z component of velocity is very small, it can safely be neglected. Recalling that $\dot{x} = v_x$ and $\dot{s} = v_s$, we obtain the equations for the motion in the $x - s$ plane:

$$\begin{aligned} \ddot{x} &= -\dot{s} \frac{e}{m_0 \gamma} B_z(s), \\ \ddot{s} &= -\dot{x} \frac{e}{m_0 \gamma} B_z(s). \end{aligned} \tag{4}$$

The horizontal motion only has a weak influence on the longitudinal velocity, and therefore:

$$\dot{x} = v_x << c \quad \text{and} \quad \dot{s} = v_s = \beta \cdot c = const. \tag{5}$$

Since in (4) only the first equation really matters, we obtain:

$$\ddot{x} = -\frac{\beta c e}{m_0 \gamma} B_0 \sum_{i=1}^{N} a_i \sin(k_i s) \ . \tag{6}$$

By inserting the expressions of the first derivatives (spatial coordinates) $\dot{x} = \beta c x'$ and $\ddot{x} = \beta^2 c^2 x''$, the following equation for x'' is obtained:

$$x'' = -\frac{e B_0}{m_0 \beta c \gamma} \sum_{i=1}^{N} a_i \sin\left(2\pi \frac{s}{\lambda_i} \right) \ . \tag{7}$$

In particular, when $\beta = 1$, we get:

$$\begin{aligned} x'(s) &= \frac{B_0}{2\pi m_0 \gamma c} \sum_{i=1}^{N} \lambda_i a_i \cos\left(2\pi \frac{s}{\lambda_i} \right), \\ x(s) &= \frac{B_0}{4\pi^2 m_0 \gamma c} \sum_{i=1}^{N} \lambda_i^2 a_i \sin\left(2\pi \frac{s}{\lambda_i} \right). \end{aligned} \tag{8}$$

Next, we define the wiggler or undulator dimensionless parameters as:

$$K_i = \frac{\lambda_i e B_0}{2\pi \cdot m_0 c}, \quad i = 1, .., \mathrm{N} \ . \tag{9}$$

The *K* parameters provide us with the distinction between wiggler and undulator. Consequently, we have:

$$\begin{aligned} &\text{undulator if } K \le 1, \\ &\text{wiggler if } K > 1 \ . \end{aligned} \tag{10}$$

If in equation (8) it is assumed that the horizontal motion is described by a constant average velocity, we obtain:

$$x'(s) = \frac{1}{\gamma} \sum_{i=1}^{N} K_i a_i \cos\left(2\pi \frac{s}{\lambda_i}\right) \ . \tag{11}$$

Using the relations: $\dot{x} = \beta c x'$ and $\omega_i = k_i \beta c$, we may write:

$$\dot{x}(t) = \frac{\beta c}{\gamma} \sum_{i=1}^{N} K_i a_i \cos(\omega_i t). \tag{12}$$

The velocity relation $\dot{s}^2 = (\beta c)^2 - \dot{x}^2$, with $\beta^2 = 1 - \frac{1}{\gamma^2}$ provides the following result:

$$\dot{s}(t) = c\sqrt{1 - \left(\frac{1}{\gamma^2} + \frac{\dot{x}^2}{c^2}\right)}. \tag{13}$$

The series expansion of the square root in the right hand side of equation (13) gives us the approximation:

$$\dot{s}(t) = c\left[1 - \frac{1}{2\gamma^2}\left(1 + \frac{\gamma^2}{c^2}\dot{x}^2\right)\right]. \tag{14}$$

Then, we have:

$$\dot{s}(t) = c\left\{1 - \frac{1}{2\gamma^2}\left[1 + \frac{\beta^2}{2}\left(\sum_{i=1}^{N} K_i a_i \cos(\omega_i t)\right)^2\right]\right\}. \tag{15}$$

Next, from equations (11) and (15) [9], we get:

$$\begin{aligned} \dot{s}(t) = {} & c\left[1 - \frac{1}{2\gamma^2}\left(1 + \frac{\beta^2}{2}\sum_{i=1}^{N} K_i^2 a_i^2\right)\right] + \\ & + \beta^2 c\left\{\frac{1}{2\gamma^2}\left[-\sum_{i=1}^{N} \frac{K_i^2 a_i^2}{2}\cos(2\omega_i t)\right]\right\} + \\ & + \beta^2 c\left\{\frac{1}{2\gamma^2}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} K_i a_i K_j a_j \left[\cos((\omega_j - \omega_i)\cdot t) - \cos((\omega_j + \omega_i)\cdot t)\right]\right\}. \end{aligned} \tag{16}$$

The above equation can be explicitly represented as a superposition of the average velocity $\langle \dot{s} \rangle$, with an oscillatory term $\Delta\dot{s}(t)$:

$$\dot{s}(t) = \langle \dot{s} \rangle + \Delta\dot{s}(t) \ , \tag{17}$$

where the average velocity $\langle \dot{s} \rangle$ is defined as:

$$\langle \dot{s} \rangle = c\left[1 - \frac{1}{2\gamma^2}\left(1 + \frac{\beta^2}{2}\sum_{i=1}^{N} K_i^2 a_i^2\right)\right] , \tag{18}$$

and the oscillatory contribution contains two terms $\Delta\dot{s}(t) = \Delta\dot{s}(t)_1 + \Delta\dot{s}(t)_2$:

$$\begin{aligned} \Delta\dot{s}(t)_1 &= \beta^2 c\left\{\frac{1}{2\gamma^2}\sum_{i=1}^{N}\left[-\frac{K_i^2 a_i^2}{2}\cos(2\omega_i t)\right]\right\}, \\ \Delta\dot{s}(t)_2 &= \beta^2 c\left\{\frac{1}{2\gamma^2}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} K_i a_i K_j a_j \left[\cos((\omega_j - \omega_i)\cdot t) - \cos((\omega_j + \omega_i)\cdot t)\right]\right\}. \end{aligned} \tag{19}$$

From equation (18), we obtain for the relative velocity β^*:

$$\beta^* = \frac{\langle \dot{s} \rangle}{c} = 1 - \frac{1}{2\gamma^2}\left(1 + \frac{1}{2}\sum_{i=1}^{N} K_i^2 a_i^2\right) . \tag{20}$$

Accordingly:

$$\begin{aligned} \dot{x}(t) &= \frac{\beta c}{\gamma}\left[\sum_{i=1}^{N} K_i a_i \cos\left(2\pi \frac{s}{\lambda_i}\right)\right], \\ \dot{s}(t) &= \beta^* c + \frac{c\beta^2}{4\gamma^2}\Delta\dot{s}(t)_1 + \frac{c\beta^2}{2\gamma^2}\Delta\dot{s}(t)_2. \end{aligned} \tag{21}$$

In the laboratory frame, the velocity can be evaluated by integration, from equation (21) with $\beta = 1$:

$$\begin{aligned} x(t) &= \frac{1}{\gamma}\left[\sum_{i=1}^{N} \frac{K_i}{k_i} a_i \sin(\omega_i t)\right], \\ s(t) &= \beta^* ct + \frac{c\beta^2}{4\gamma^2}\int \Delta\dot{s}(t)_1 dt + \frac{c\beta^2}{2\gamma^2}\int \Delta\dot{s}(t)_2 dt. \end{aligned} \tag{22}$$

When the Lorentz transformations,

$$x^* = \mathrm{x} \quad \text{and} \quad s^* = \gamma(s - \beta ct) , \tag{23}$$

are used in equation (22), the particle motion, as observed in the frame K^* of the mass center, is obtained. With respect to the laboratory frame, the system moves with velocity β^*. The radiation emitted in the undulator for the periodic motion and frequency Ω_w in the laboratory frame is:

$$\Omega_w = k_u \beta c . \tag{24}$$

In a moving frame with velocity β^*, the transformed frequency ω^* reads:

$$\omega^* = \gamma^* \Omega_w . \tag{25}$$

For a photon emitted under angle Θ_0 in the laboratory frame, by using the well-known relations: energy $E = \hbar\omega$, photon momentum $p = \hbar\omega / c$, and the 4-vector algebra, we obtain for the frequency ω_w:

$$\omega_w = \frac{\omega^*}{\gamma^*\left(1-\beta^*\cos\Theta_0\right)} \quad . \tag{26}$$

At small values of Θ_0, $\cos\Theta_0$ is well approximated by $1-\Theta_0^2/2$, and from equation (20) we obtain the following coherence condition for the undulator radiation:

$$\lambda_w = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{1}{2}\sum_{i=1}^{N}K_i^2 a_i^2+\gamma^2\Theta_0^2\right) \quad . \tag{27}$$

Conclusion

The three frequency model, proposed by us [4] and improved by Tripathi and Mishra [5], was here developed and extended to N-frequency. All equations describing the beam dynamics (equations (6) to (22)) were generalized to N frequencies. This represents a new point of view, more general, in the approach of the dynamics of the phenomenon. Analytical expressions for x and s trajectory components were obtained. In s, we have *sinusoidal* harmonics with frequencies: $2\omega_i$ (i = 1 .. N) and $\omega_j \pm \omega_i$ (i = 1 .. N-1, $j = i$+1 .. N). Also, we derived a general expression for the average velocity of electrons, β^*. The results show that the radiation frequency depends on the N harmonics in magnetic field components, described by K dimensionless parameters. The expressions maintain their validity for N fractional frequencies. More importantly, the above derived expressions can be extended to integer, and also to fractional frequency combinations. As future work, we intend to study also the contribution of the other transversal magnetic field component.

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Ondulator cu N frecvenţe

Rezumat

Laserii cu electroni liberi (FEL) implică elaborarea de dispozitive de inserţie.Generarea radiaţiei sincrotronice foloseşte inele de acumulare şi componente ale laserilor cu electroni liberi. In acest tip de dispozitive de inserţie mişcarea electronilor relativişti generează radiaţie coerentă acordabilă cu o lungime de undă dorită. Un fascicul de electroni relativişti suferă o oscilaţie transversală şi emite radiaţie. În cercetarea şi dezvoltarea FELuna din tendinţele principale este elaborarea de dispozitive compacte. Ondulatorul este componenta principală în care are loc fenomenul de radiaţie coerentă. Calitatea fasciculului şi a radiaţiei rezultante poate fi îmbunătăţită încercând diferite linii de câmp, paşi de înfăşurare, dimensiuni (tuguiere). Modelele s-au dezvoltat de la schema longitudinală cu o singură frecvenţă la cea transversală cu două frecvenţe, apoi la cea transversală cu trei frecvenţe şi acum la modelul cu formă generală cu N frecvenţe. Această dezvoltare de la o singură frecvenţă la frecvenţe multiple răspunde unei necesităţi practice. Multe aplicaţii, în domenii diferite precum biologia, farmacia, chimia, metamaterialele şi altele, necesită mai multe fascicule cu diferite frecvenţe pentru a obţine interacţii selective cu materia. Se prezintă o nouă structură magnetică de ondulator pentru laserii cu electroni liberi. Pentru ondulatorul cu N frecvenţe şi injecţie de-a lungul axei longitudinale se calculează traiectoria electronilor. Această structură reduce valoarea de vârf a componentei câmpului magnetic.